Correction exercices Supplémentaires.

Exo 1

1) $S = \sum_{n \geq 0} \frac{7n^2+5}{n^3+2}$.

$u_n \underset{\infty}{\sim} \frac{7n^2}{n^3} = \frac{7}{n}$ et $\sum \frac{1}{n}$ DV de plus $\frac{7}{n} > 0$ donc S DV

2) $S = \sum_{n \geq 1} \left(\frac{3}{n}\right)^n$. Cauchy $u_n^{1/n} = \frac{3}{n} \xrightarrow[n \to +\infty]{} 0 < 1$

S CV

3) $S = \sum_{n \geq 0} u_n$ $u_n = \begin{cases} 1/3^k \\ 2/3^{k+1} \end{cases}$

Si $n = 2k$ $\frac{u_{n+1}}{u_n} = \frac{2}{3^{k+1}} \times \frac{3^k}{1} = \frac{2}{3} \longrightarrow \frac{2}{3} < 1$ $\Big]$ d'Alembert

Si $n = 2k+1$ $\frac{u_{n+1}}{u_n} = \frac{1}{3^{k+1}} \times \frac{3^{k+1}}{2} = \frac{1}{2} \longrightarrow \frac{1}{2} < 1$ $\Big\}$ $\sum u_n$ CV

4) $u_n = \left(\frac{2n+1}{5n+4}\right)^n$ $\quad u_n^{1/n} = \frac{2n+1}{5n+4} \longrightarrow \frac{2}{5} < 1$

$\sum u_n$ CV d'après Cauchy

Exo 2

1) $I = \int_2^5 \frac{x}{(2x^2-3)^4}\,dx = \int_2^5 x\,(2x^2-3)^{-4}\,dx$

$= \left[-\frac{1}{12}(2x^2-3)^{-3}\right]_2^5 = -\frac{1}{12}\left(47^{-3} - 5^{-3}\right)$

$$2)\ \int_2^4 (x^2+x)e^{2x^3+3x^2+1}\,dx = \left[\frac{1}{6}e^{2x^3+3x^2+1}\right]_2^4 = \frac{e^{177}-e^{29}}{6}$$

Exo 3

$$v_m = \frac{1}{7-3}\int_3^7 (x-2)e^{x+1}\,dx = \frac{1}{4}\left(\left[(x-2)e^{x+1}\right]_3^7 - \int_3^7 e^{x+1}\,dx\right)$$

IPP

$u' = e^{x+1}$ $\qquad u = e^{x+1}$

$v = x-2$ $\qquad v' = 1$

$$v_m = \frac{1}{4}\left(5e^8 - e^4 - \left[e^{x+1}\right]_3^7\right) = e^8.$$

Exo 4

$$I = \int_1^e 1 \ln^2(x)\,dx = \left[x \ln^2 x\right]_1^e - \int_1^e 2 \ln x \,dx$$

IPP

$u = \ln^2 x$ $\qquad u' = 2\,\frac{\ln x}{x}$

$v' = 1$ $\qquad v = x$

$$I = e - 2\int_1^e 1 \ln x\,dx = e - 2\left(\left[x \ln x\right]_1^e - \int_1^e \frac{x}{x}\,dx\right)$$

IPP

$u = \ln x$ $\qquad u' = 1/x$

$v' = 1$ $\qquad v = x$

$$I = e - 2\left(e - [x]_1^e\right)$$

$$I = e - 2(e - e + 1) = e - 2.$$

Exo 4

$$I = \int_2^5 \frac{x^2}{x^3+4}\,dx$$

chgt variable $y = x^3$

$$dy = 3x^2\,dx$$

$x = 2$ $y = 8$

$x = 5$ $y = 125$

$$I = \int_8^{125} \frac{dy}{3(y+4)} = \frac{1}{3}\Big[\ln|y+4|\Big]_8^{125}$$

$$= \frac{1}{3}\left(\ln 129 - \ln 12\right).$$